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A photon rest mass and the propagation of longitudinal electric waves in interstellar and intergalactic space

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Abstract. This paper deals with the effect of a nonzero photon rest mass m on the propagation of longitudinal electric waves in a plasma for which the electrons are treated as a warm fluid with scalar pressure. The Proca equations are used to obtain a dispersion relation which describes two longitudinal wave types; when $m = 0$, one wave type reduces to the usual electron acoustic wave while the second type, the Proca wave, ceases to exist. For frequencies away from resonance, the cold plasma model is approximately valid. In this paper, dispersion and absorption are discussed for frequencies both away from and near to resonance; particular attention is paid to Proca waves, and the possibilities of their travelling over interstellar and intergalactic distances are assessed.

1. Introduction

In the last few years considerable attention has been given to the question of the rest mass m of the photon. In particular, a number of authors has considered the dispersion of transverse electromagnetic waves which would arise from a nonzero m (eg Gintsburg 1964, Goldhaber and Nieto 1971, Kobzarev and Okun' 1968); the possibility of detecting such dispersion in radiation of cosmic origin, in the presence of dispersion arising from the interstellar (Lee 1971, Cole 1972) or intergalactic (Burman 1972a) plasma, has been discussed. Gertsenshtein (1971) has suggested that the events detected by Weber might be caused by longitudinal electric waves. In a cold plasma, as in free space, longitudinal electric waves can propagate if $m \neq 0$; dispersion and absorption of such waves in interstellar space have been discussed previously (Burman 1972b,c). That work is extended here to allow for a nonzero plasma temperature; this enables propagation in the vicinity of resonance, where the cold plasma model fails, to be investigated. Propagation in both interstellar and intergalactic space is discussed.

2. The dispersion relation

A nonzero m can be incorporated into electromagnetic theory by replacing Maxwell's equations by their simplest relativistic generalization, namely the Proca equations. Let $c \equiv (\mu_0\epsilon_0)^{-1/2}$, μ_0 and ϵ_0 being the permeability and permittivity of free space, and write m as $\hbar\omega_c/c^2$, $\hbar$ being Planck's constant divided by 2π. In MKS units, Proca's equations lead to the wave equations

$$\left(\Box^2 - \frac{\omega_c^2}{c^2}\right)\phi = -\frac{\rho}{\epsilon_0} \tag{1}$$

and

$$\left(\Box^2 - \frac{\omega_c^2}{c^2}\right)A = -\mu_0 J \tag{2}$$

for the scalar and vector potentials ϕ and A; the net charge and current densities are denoted by ρ and J.

Suppose that the fields are small-amplitude disturbances in a plasma which is stationary in the unperturbed state and has unperturbed electron number density N, corresponding to an angular plasma frequency ω_p; ion motion will be neglected. Let e and m_e denote the charge and mass of an electron, ν the effective electron–heavy particle collision frequency and $\boldsymbol{v}$ the electron fluid velocity. A time factor $e^{i\omega t}$ will be taken. In the linear approximation $\boldsymbol{J} = Ne\boldsymbol{v}$ and, from conservation of charge, $\rho = (ie/\omega)\nabla \cdot (N\boldsymbol{v})$. Suppose that the plasma is homogeneous and isotropic: there will be no coupling between longitudinal and transverse waves; the former have an electric field $\boldsymbol{E}$, but no magnetic field.

If n denotes the refractive index, then (1) and (2) give

$$\left(1 - n^2 - \frac{\omega_c^2}{\omega^2}\right)\phi = -cn\frac{m_e}{e}\frac{\omega_p^2}{\omega^2}v_l \tag{3}$$

and

$$\left(1 - n^2 - \frac{\omega_c^2}{\omega^2}\right)\boldsymbol{A} = -\frac{m_e}{e}\frac{\omega_p^2}{\omega^2}\boldsymbol{v} \tag{4}$$

where v_l is the longitudinal component of $\boldsymbol{v}$. Since $\boldsymbol{E} = -\nabla\phi - \partial \boldsymbol{A}/\partial t$, it follows from (3) and (4) that

$$\boldsymbol{E} = \frac{i\omega m_e}{e}\frac{\omega_p^2}{\omega^2}\left(\frac{\boldsymbol{v} - n^2\boldsymbol{v}_l}{1 - n^2 - \omega_c^2/\omega^2}\right). \tag{5}$$

If the electrons are represented as a warm fluid with scalar pressure perturbations p, then the linearized equation of motion is

$$i\omega m_e N\left(1 - \frac{i\nu}{\omega}\right)\boldsymbol{v} = eN\boldsymbol{E} - \nabla p. \tag{6}$$

Combining the linearized equation of state and the linearized equation of continuity gives (Friedlander 1958)

$$i\omega p + m_e N u^2 \nabla \cdot \boldsymbol{v} = 0 \tag{7}$$

where u is the acoustic speed in the electron gas. Elimination of p between (6) and (7) leads to

$$\left(\frac{i\omega e}{m_e}\right)\boldsymbol{E} + \omega^2\left(1 - \frac{i\nu}{\omega}\right)\boldsymbol{v} + u^2\nabla\nabla \cdot \boldsymbol{v} = 0. \tag{8}$$

Equations (5) and (8) show that propagation of a transverse wave is unaffected by a nonzero temperature, while, for longitudinal waves

$$\frac{u^2}{c^2}n^4 - \left\{1 - \frac{i\nu}{\omega} - \frac{\omega_p^2}{\omega^2} + \frac{u^2}{c^2}\left(1 - \frac{\omega_c^2}{\omega^2}\right)\right\}n^2 + \left(1 - \frac{i\nu}{\omega}\right)\left(1 - \frac{\omega_c^2}{\omega^2}\right) - \frac{\omega_p^2}{\omega^2} = 0. \tag{9}$$

Equation (9), regarded as a quadratic in n^2, describes two longitudinal wave types.

If $m = 0$, so that $\omega_c = 0$, equation (9) reduces to

$$\frac{u^2}{c^2}n^2 = 1-\frac{i\nu}{\omega}-\frac{\omega_p^2}{\omega^2} \tag{10}$$

together with the spurious root $n^2 = 1$, there is only one longitudinal wave type present—the usual electron acoustic wave. If $u = 0$, then (10) becomes an equation for ω, describing the usual nonpropagating plasma oscillation.

If $u = 0$, corresponding to a cold plasma, then (9) gives

$$n^2 = 1-\frac{\omega_c^2/\omega^2}{1-(\omega_p^2/\omega^2)(1-i\nu/\omega)^{-1}}. \tag{11}$$

If $\nu = 0$, then n is zero when $\omega = (\omega_p^2+\omega_c^2)^{1/2}$ and is infinite when $\omega = \omega_p$. In an actual plasma, thermal effects and damping prevent n from becoming infinite.

If u^2/c^2 is small but not zero, then (9) gives

$$n^2 \simeq \frac{c^2}{u^2}\theta+\frac{\omega_c^2\omega_p^2}{\omega^4\theta}\left[1+\frac{u^2}{c^2\theta^2}\left\{\theta-\frac{\omega_c^2}{\omega^2}\left(1-\frac{i\nu}{\omega}\right)\right\}\right] \tag{12a}$$

or

$$n^2 \simeq 1-\frac{\omega_c^2}{\omega^2\theta}\left[1-\frac{i\nu}{\omega}+\frac{u^2}{c^2\theta^2}\frac{\omega_p^2}{\omega^2}\left\{\theta-\frac{\omega_c^2}{\omega^2}\left(1-\frac{i\nu}{\omega}\right)\right\}\right] \tag{12b}$$

where $\theta \equiv 1-i\nu/\omega-\omega_p^2/\omega^2$. Equation (12*a*) corresponds to the usual electron acoustic wave in a warm plasma, but is modified by the nonzero photon rest mass. Equation (12*b*) corresponds to the wave type considered previously (Burman 1972b,c) in the case of a cold plasma, but incorporates a correction for thermal effects. If collisions are neglected, then (12) fails when ω is near ω_p.

The wave type whose existence depends on a nonzero value of m will be referred to as the Proca wave; the other wave type will be referred to as the Langmuir wave.

3. Dispersion

This section deals with the dispersion relation (9) for the case in which collisions are negligible. Thus

$$\frac{u^2}{c^2}n^4-\left(1+\frac{u^2}{c^2}\right)\left(1-\frac{\omega_1^2}{\omega^2}\right)n^2+1-\frac{\omega_2^2}{\omega^2} = 0 \tag{13}$$

where

$$\omega_1^2 \equiv \frac{\omega_p^2+\omega_c^2u^2/c^2}{1+u^2/c^2} \tag{14a}$$

and

$$\omega_2^2 \equiv \omega_p^2+\omega_c^2. \tag{14b}$$

It should be noted that ω_2^2 always exceeds ω_1^2. The discriminant of the quadratic equation (13) is positive: the two values of n^2 are always real and distinct.

For large ω,

$$n^2 = \frac{c^2}{u^2}\left(1-\frac{\omega_p^2}{\omega^2}\right)+O(\omega^{-4}) \qquad \text{or} \qquad 1-\frac{\omega_c^2}{\omega^2}+O(\omega^{-4}). \tag{15}$$

When $\omega = \omega_2$,

$$n^2 = \left(\frac{c^2}{u^2}+1\right)\left(1-\frac{\omega_1^2}{\omega_2^2}\right) \qquad \text{or} \qquad 0. \tag{16}$$

When $\omega = \omega_1$,

$$n^2 = \pm\frac{c}{u}\left(\frac{\omega_2^2}{\omega_1^2}-1\right)^{1/2}. \tag{17}$$

For small ω,

$$n^2 = \frac{\omega_p^2+\omega_c^2}{\omega_p^2+\omega_c^2 u^2/c^2}+O(\omega^2) \qquad \text{or} \qquad \frac{c^2}{u^2}\left(1-\frac{\omega_p^2}{\omega^2}\right)-\frac{\omega_c^2}{\omega^2}-\frac{(1-u^2/c^2)\omega_c^2}{\omega_p^2+\omega_c^2u^2/c^2}+O(\omega^2). \tag{18}$$

In a plot of n^2 against ω^2, the two branches never cross. One branch crosses the ω^2 axis and is negative, corresponding to evanescent disturbances, for $\omega < \omega_2$; the other branch remains positive, corresponding to propagating waves, for all frequencies. The upper branch represents the Langmuir wave when $\omega \gg \omega_p$ and the Proca wave when $\omega \ll \omega_p$; the lower branch represents the Proca wave when $\omega \gg \omega_p$ and the evanescent Langmuir wave when $\omega \ll \omega_p$.

The frequency interval (ω_1, ω_2) will be referred to as the resonance interval. In or near this interval, if u^2/c^2 is very small, then

$$\frac{u^2}{c^2}n^2 \simeq 1-\frac{\omega_1}{\omega}\pm\left\{\left(1-\frac{\omega_1}{\omega}\right)^2+2\frac{u^2}{c^2}\left(\frac{\omega_2}{\omega}-1\right)\right\}^{1/2} \tag{19}$$

where the upper and lower signs correspond respectively to the propagating wave and the wave which is evanescent when $\omega < \omega_2$. A region of an inhomogeneous plasma in which the frequency is in the resonance interval will be referred to as a resonance region.

4. Absorption away from resonance

This section deals with absorption of the Proca wave for frequencies away from resonance; the cold plasma dispersion relation is adequate.

Suppose that ω_c^2/ω^2 and ν/ω are small and that ω is not close to ω_p. Then, from (11), if $-k$ denotes the imaginary part of n,

$$k \simeq \frac{\nu}{2\omega}\frac{\omega_c^2\omega_p^2}{(\omega^2-\omega_p^2)^2}. \tag{20}$$

If $\omega^2 \ll \omega_p^2$, then

$$\frac{\omega k}{c} \simeq \frac{\nu}{2c}\frac{\omega_c^2}{\omega_p^2}, \tag{21}$$

which quantity is independent of frequency. If $\omega^2 \gg \omega_p^2$, then

$$\frac{\omega k}{c} \simeq \frac{\nu}{2c}\frac{\omega_c^2\omega_p^2}{\omega^4}, \tag{22}$$

which is strongly dependent on frequency.

In a plasma consiting of electrons and protons of equal number densities N, and with equal electron and ion temperatures T, the effective electron–proton collision frequency is given by (Ginzburg 1964 p 403)

$$\nu = 5{\cdot}5\frac{N}{T^{3/2}}\ln\left(220\frac{T}{N^{1/3}}\right) \tag{23a}$$

if $T \ll 3\times10^5$ K and by

$$\nu = 5{\cdot}5\frac{N}{T^{3/2}}\ln\left(10^4\gamma\frac{T^{2/3}}{N^{1/3}}\right) \tag{23b}$$

if $T \gg 3\times10^5$ K; γ is a number of order unity and N is in particles per cm^3 (so that $\omega_p/2\pi \simeq 9\times10^3N^{1/2}$). For $T \sim 3\times10^5$ K, both formulae give reasonably accurate results (Ginzburg 1964 p 403).

Using (23), equations (21) and (22) show that, apart from logarithmic factors, $\omega k/c \propto \omega_c^2/T^{3/2}$ and $\omega k/c \propto \omega_c^2N^2/\omega^4T^{3/2}$ respectively: attenuation decreases rapidly with increasing T; it is almost independent of N when $\omega^2 \ll \omega_p^2$ and increases rapidly with increasing N when $\omega^2 \gg \omega_p^2$.

4.1. Interstellar absorption

The absorption of Proca waves in interstellar space has been discussed previously (Burman 1972c). It can be added that if there exists a medium with $T_e \sim 10^4$ K in which the cool HI regions are embedded (Clark 1965, Field 1969), then, since $\nu/N_e \sim 10^{-4}$ cm^{3}s^{-1} in the intercloud medium, it follows that most of the absorption will occur in the cool regions: the previous estimates of absorption over galactic distances will have to be multiplied by that fraction, perhaps around 10^{-1}, of the path which passes through cool regions.

4.2. Intergalactic absorption

The absence of Lyman α absorption in the observed continuous spectra of some quasars with red shifts z of about 2 implies, if the red shift is cosmological, that the number density of atomic hydrogen N_H in intergalactic space is less than about 3×10^{-11} cm^{-3} at z near 2, corresponding, if the degree of ionization remains constant, to $N_H \lesssim 10^{-12}$ cm^{-3} at the present epoch (Sciama 1971a p 134). A similar argument shows that the number density of molecular hydrogen in intergalactic space is less than about 6×10^{-10} cm^{-3} at the present epoch (Peebles 1971 p 98). Thus the intergalactic medium could be highly ionized hydrogen. Since a cold plasma would rapidly recombine and since most ionizing mechanisms heat the gas, the hydrogen would be likely to be at a temperature exceeding 10^5 K (Sciama 1971a p 140). If the hydrogen density is near the cosmologically critical value, and if the ionization is that associated with the temperature T, then $T > 10^6$ K; furthermore, temperatures much greater than 10^6 K would lead, through bremsstrahlung, to a kilovolt x ray flux in excess of that observed (Sciama 1971b p 190).

A model with the critical density and with $T \sim 10^6$ K at z near 2, corresponding to $T \sim 3 \times 10^5$ K at the present epoch, appears to be compatible with observations (Sciama 1971b p 191); with this model, $\omega_p/2\pi \sim 30\,\mathrm{s}^{-1}$ and $\nu \sim 10^{-11}\mathrm{s}^{-1}$ at the present epoch. Nearby intergalactic gas might be less highly ionized (Sciama 1971b p 191).

With $\omega_p/2\pi \sim 30\,\mathrm{s}^{-1}$ and $\nu \sim 10^{-11}\mathrm{s}^{-1}$, since $\omega_c/2\pi < \frac{1}{2}\mathrm{s}^{-1}$ (Goldhaber and Nieto 1971), the right hand sides of (21) and (22) are $\lesssim 10^{-1}\,\mathrm{Mpc}^{-1}$ and $\lesssim 10^5/f^4\,\mathrm{Mpc}^{-1}$, where f is in inverse seconds. Consider propagation over the linear dimension D of the local supercluster, namely 10 Mpc. If f is well below $30\,\mathrm{s}^{-1}$, then $\omega k D/c \lesssim 1$; absorption will be significant if m is close to its established upper limit (Goldhaber and Nieto 1971). If f is well above $30\,\mathrm{s}^{-1}$, then $\omega k D/c \lesssim 10^6/f^4$; absorption is very light.

For propagation over sufficiently large cosmological distances allowance must be made for the variation of the wave frequency, relative to comoving observers, as the wave propagates; if the universe is evolving, allowance must also be made for the variations of N and T with epoch.

5. Absorption in warm plasmas

In this section, the dispersion relation (9) is investigated for small values of Z, which denotes ν/ω. The results are used to treat absorption in or near a resonance interval.

Let a subscript 0 denote a quantity evaluated when $Z = 0$. The dispersion relation (9) can be written

$$\left(\frac{u^2}{c^2}\right)n^4 - bn^2 + c = 0 \tag{24}$$

where $b = b_0 - \mathrm{i}Z$ and $c = c_0 - \mathrm{i}Z(1-\omega_c^2/\omega^2)$. As a quadratic in n^2, equation (24) has a discriminant Δ which is given by

$$\Delta = \Delta_0 - 2\mathrm{i}Z\left(\Delta_0 - 4\frac{u^2}{c^2}\frac{\omega_c^2\omega_p^2}{\omega^4}\right)^{1/2} + Z^2.$$

As mentioned in § 4, Δ_0 never vanishes. Hence, if terms in Z^2 are neglected, then

$$n^2 \simeq n_0^2 - \frac{\mathrm{i}Z}{2}\frac{c^2}{u^2}\left\{1 \mp \left(1 - \frac{4}{\Delta_0}\frac{u^2}{c^2}\frac{\omega_c^2\omega_p^2}{\omega^4}\right)^{1/2}\right\}. \tag{25}$$

Write $n \equiv n_r - \mathrm{i}k$ where n_r and k are real. Since $\Delta_0 \geqslant 4(u^2/c^2)\omega_c^2\omega_p^2/\omega^4$, equation (25) shows that, to first order in Z, $n_r \simeq n_0$ and

$$k \simeq \frac{Z}{4n_0}\frac{c^2}{u^2}\left\{1 \mp \left(1 - \frac{4}{\Delta_0}\frac{u^2}{c^2}\frac{\omega_c^2\omega_p^2}{\omega^4}\right)^{1/2}\right\}. \tag{26}$$

For the upper sign in (26), $kn_0 \to 0$ as $\omega_c \to 0$: the upper sign corresponds to the Proca wave as defined in § 2.

The discriminant Δ_0 is given exactly by the equation

$$\Delta_0 = \left(1 + \frac{u^2}{c^2}\right)^2\left(1 - \frac{\omega_1^2}{\omega^2}\right)^2 - 4\frac{u^2}{c^2}\left(1 - \frac{\omega_2^2}{\omega^2}\right). \tag{27}$$

If $\omega^2 \ll \omega_1^2$ and $u^2/c^2 \ll 1$, then $\Delta_0 \simeq (\omega_1/\omega)^4$. If $\omega^2 \gg \omega_2^2$ and $u^2/c^2 \ll 1$, then $\Delta_0 \simeq 1$. If $u^2/c^2 \ll 1$ and $\omega_c^2/\omega_p^2 \ll 1$, then $\omega_1/\omega_p \simeq 1 - u^2/2c^2$ and $\omega_2/\omega_p \simeq 1 + \omega_c^2/2\omega_p^2$.

If $\omega_c^2 u^2/\omega_p^2 c^2 \ll 1$, then

$$\frac{\omega_2^2}{\omega_1^2} \simeq 1+\frac{u^2}{c^2}+\frac{\omega_c^2}{\omega_p^2}. \tag{28}$$

From (27) and (28), if $\omega_c^2/\omega_p^2 \ll u^2/c^2 \ll 1$, then $\Delta_0 \simeq 4u^4/c^4$, $9u^4/4c^4$ and u^4/c^4 when $\omega = \omega_1, \frac{1}{2}(\omega_1+\omega_2)$ and ω_2, respectively, suggesting that Δ_0 can be approximated by $2u^4/c^4$ in or near the resonance interval.

If

$$\frac{4}{\Delta_0}\frac{u^2}{c^2}\frac{\omega_c^2\omega_p^2}{\omega^4} \ll 1, \tag{29}$$

then (26) gives

$$\frac{\omega k}{c} \simeq \frac{\nu}{2n_0 c}\frac{\omega_c^2\omega_p^2}{\omega^4\Delta_0} \tag{30a}$$

or

$$\frac{\omega k}{c} \simeq \frac{\nu}{2n_0 c}\left(\frac{c^2}{u^2}-\frac{\omega_c^2\omega_p^2}{\omega^4\Delta_0}\right). \tag{30b}$$

For $u^2/c^2 \ll 1$, equation (30*a*) reduces to (21) when $\omega^2 \ll \omega_1^2$ and to (22) when $\omega^2 \gg \omega_2^2$, apart from the presence of n_0 which is near unity in the former case if $\omega_c^2 \ll \omega_p^2$ and in the latter case if $\omega_c^2 \ll \omega^2$.

If $\omega_c^2/\omega_p^2 \ll u^2/c^2 \ll 1$, then use of (17), (13), (16) and (28) shows that when $\omega = \omega_1$,

$$n^2 \simeq \pm\left(1+\frac{\omega_c^2/\omega_p^2}{2u^2/c^2}\right), \tag{31a,b}$$

when $\omega = \frac{1}{2}(\omega_1+\omega_2)$,

$$n^2 \simeq 1+\frac{2\omega_c^2/\omega_p^2}{3u^2/c^2} \quad \text{or} \quad -\frac{1}{2}-\frac{\omega_c^2/\omega_p^2}{6u^2/c^2}, \tag{32a, b}$$

and when $\omega = \omega_2$,

$$n^2 \simeq 1+\frac{u^2}{c^2}+\frac{\omega_c^2/\omega_p^2}{u^2/c^2} \quad \text{or} \quad n^2 = 0. \tag{33a, b}$$

Suppose that $\omega_c^2/\omega_p^2 \ll u^2/c^2 \ll 1$ and consider frequencies in or near the resonance interval. For the wave which propagates when $\omega < \omega_2$, equations (31) to (33) suggest that n_0 can be approximated by one. Also, (29) is satisfied and (30*a*) gives

$$\frac{\omega k}{c} \simeq \frac{\nu}{4c}\frac{\omega_c^2}{\omega_p^2}\frac{c^4}{u^4}. \tag{34}$$

Using (23), equation (34) shows that, apart from a logarithmic factor, $\omega k/c \propto \omega_c^2/T^{7/2}$:attenuation decreases very rapidly with increasing T, but is almost independent of N.

Suppose that the plasma density varies with distance in a certain direction and consider a Proca wave travelling in that direction and meeting a resonance region. If the wave is incident from a region in which $\omega \gg \omega_2$, then the wave will be reflected since, for the branch concerned, n vanishes when $\omega = \omega_2$ and is imaginary when $\omega < \omega_2$; if the resonance region is of finite thickness, then some transmission will occur. If the wave is incident from a region in which $\omega \ll \omega_1$, then the wave will continue to propagate through the resonance region. In this case, if the plasma density continues to fall with distance along the wave's path, then the wave will eventually propagate as a Langmuir wave. If, at any stage, the plasma density ceases to decrease and rises back to a level for which $\omega \ll \omega_1$ again, then the wave will return to being a Proca wave; equation (34) gives the absorption suffered by the wave while passing through the resonance region, and some numerical estimates of such absorption will now be made.

5.1. Interstellar absorption

Taking $N_e \sim 10^{-3}\,\mathrm{cm}^{-3}$ and $T_e \sim 10\,\mathrm{K}$, corresponding to $\omega_p/2\pi \sim 3\times10^2\,\mathrm{s}^{-1}$, $\nu \sim 2\times10^{-3}\,\mathrm{s}^{-1}$ and $u^2/c^2 \sim 10^{-9}$, the condition $\omega_c^2/\omega_p^2 \ll u^2/c^2 \ll 1$ will be satisfied provided ω_c is at least two powers of ten below its established upper limit. Equation (34) gives $\omega k/c \sim 3\times10^{17} f_c^2\,\mathrm{pc}^{-1}$ where $f_c \equiv \omega_c/2\pi$ and is in inverse seconds.

Taking $N_e \sim 10^{-2}\,\mathrm{cm}^{-3}$ and $T_e \sim 10^2\,\mathrm{K}$, corresponding to $\omega_p/2\pi \sim 10^3\,\mathrm{s}^{-1}$, $\nu \sim 5\times10^{-4}\,\mathrm{s}^{-1}$ and $u^2/c^2 \sim 10^{-8}$, the condition $\omega_c^2/\omega_p^2 \ll u^2/c^2 \ll 1$ will be satisfied provided ω_c is at least a power of ten below its established upper limit. Equation (34) gives $\omega k/c \sim 3\times10^{13} f_c^2\,\mathrm{pc}^{-1}$ where f_c is in inverse seconds.

If a hot intercloud medium exists, with $T_e \sim 10^4\,\mathrm{K}$, $\nu/N_e \sim 10^{-4}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ and $u^2/c^2 \sim 10^{-6}$, then (34) gives $\omega k/c \sim 3\times10^7 f_c^2\,\mathrm{pc}^{-1}$ where f_c is in inverse seconds.

Resonance regions will be quite thin, but the above results suggest that absorption could be heavy unless m is some powers of ten below its established upper limit.

5.2. Intergalactic absorption

With $N_e \sim 10^{-5}\,\mathrm{cm}^{-3}$ and $T \sim 3\times10^5\,\mathrm{K}$, corresponding to $\omega_p/2\pi \sim 30\,\mathrm{s}^{-1}$, $\nu \sim 10^{-11}\,\mathrm{s}^{-1}$ and $u^2/c^2 \sim 3\times10^{-5}$, the condition $\omega_c^2/\omega_p^2 \ll u^2/c^2 \ll 1$ will be satisfied if f_c is a power of ten below its established upper limit. The approximation (34) gives $\omega k/c \sim 3\times10^2 f_c^2\,\mathrm{pc}^{-1}$ where f_c is in inverse seconds.

6. Phase and group speeds

Let v_p, which is equal to c/n, and v_g, which is given by

$$\frac{c}{v_g} = n\left(1+\frac{\omega^2}{n^2}\frac{\mathrm{d}(n^2)}{\mathrm{d}(\omega^2)}\right), \tag{35}$$

denote the phase and group speeds respectively. This section will deal with v_p and v_g for the Proca wave; collisions will be neglected.

6.1. Away from resonance

For the Proca wave well away from resonance

$$n^2 \simeq \frac{\omega^2-\omega_2^2}{\omega^2-\omega_p^2}. \tag{36}$$

For frequencies well above resonance $n < 1$ so that $v_p > c$. For frequencies well below resonance $n > 1$ so that $v_p < c$: Landau damping could arise in the kinetic theory of propagation.

With (36), equation (35) gives

$$\frac{v_g}{c} \simeq \frac{(\omega^2-\omega_p^2)^{3/2}(\omega^2-\omega_2^2)^{1/2}}{(\omega^2-\omega_p^2)^2+\omega_p^2\omega_c^2}. \tag{37}$$

For frequencies well above resonance,

$$\frac{v_g}{c} \simeq \left(1+\frac{\omega_c^2\omega_p^2}{\omega^4}\right)^{-1}. \tag{38}$$

For frequencies well below resonance,

$$\frac{v_g}{c} \simeq \frac{\omega_p}{\omega_2} \tag{39}$$

which is independent of frequency.

6.2. *Warm plasmas*

From (13),

$$\pm\Delta^{1/2}\frac{\omega^2}{n^2}\frac{\mathrm{d}(n^2)}{\mathrm{d}(\omega^2)} = (1-n^{-2})\frac{\omega_p^2}{\omega^2}+\left(\frac{u^2}{c^2}-n^{-2}\right)\frac{\omega_c^2}{\omega^2}. \tag{40}$$

Suppose that $\omega_c^2/\omega_p^2 \ll u^2/c^2 \ll 1$. For the wave which propagates when $\omega < \omega_2$, equations (31) to (33) and the values of Δ given in the last section, together with (40) and (35), show that when $\omega = \omega_1$,

$$\frac{c}{v_g} \simeq 1+\frac{\omega_c^2/\omega_p^2}{4u^4/c^4}, \tag{41}$$

when $\omega = \frac{1}{2}(\omega_1+\omega_2)$,

$$\frac{c}{v_g} \simeq 1+\frac{4\omega_c^2/\omega_p^2}{9u^4/c^4}, \tag{42}$$

and when $\omega = \omega_2$,

$$\frac{c}{v_g} \simeq 2+\frac{\omega_c^2/\omega_p^2}{u^4/c^4}. \tag{43}$$

Thus if $u^2/c^2 \ll 1$ and $\omega_c^2/\omega_p^2 \ll u^4/c^4$, then v_g varies from slightly under c when $\omega = \omega_1$ and when $\omega = \frac{1}{2}(\omega_1+\omega_2)$ to slightly under $\frac{1}{2}c$ when $\omega = \omega_2$.

6.3. *Time delay*

Consider a pulse of radiation of the Proca wave, propagating from 0 to L of a linear coordinate l. The propagation times for the frequencies in the pulse are given by

$$T = \int_0^L \frac{\mathrm{d}l}{v_g}. \tag{44}$$

If, at all points of the path, ω is well below resonance, then (39) holds and there is little dispersion.

If, at all points of the path, ω is well above resonance, then (38) holds and so, allowing the plasma to be inhomogeneous,

$$T = \frac{L}{c} + \frac{f_c^2}{cf^4}\int_0^L f_p^2\,\mathrm{d}l \tag{45}$$

where $f_p \equiv \omega_p/2\pi$; hence

$$\frac{\mathrm{d}T}{\mathrm{d}f} = \frac{-4f_c^2}{cf^5}\int_0^L f_p^2\,\mathrm{d}l. \tag{46}$$

If Proca waves are discovered and are found to undergo dispersion, then use of (46) will allow an evaluation of the product of m^2 with the integrated electron density along the path.

7. Concluding remarks

This paper has dealt with the dispersion and absorption of longitudinal electric waves in interstellar and intergalactic space when the photon rest mass m is nonzero. The plasma electrons have been treated as a warm fluid with scalar pressure; ion motion has been neglected. Attention has been focused on the waves, referred to as Proca waves, which exist only if m is nonzero. These waves, if they exist at all, will be only weakly coupled to matter (Bass and Schrödinger 1955), which will make them very difficult to detect. If they do exist, Proca waves could travel across cosmic distances for much lower frequencies than in the case of transverse waves: in a warm loss-free plasma, the latter, like Langmuir waves, are cut off for $\omega < (\omega_p^2 + \omega_c^2)^{1/2}$, whereas it has been shown here that, for Proca waves, propagation is possible at all frequencies. Proca waves could be reflected or heavily absorbed in regions where the wave frequency is near the plasma frequency. Observations of Proca waves could provide information on the photon rest mass and on the interstellar or intergalactic plasma.

Future theoretical work should allow for ion motion and the effect of a magnetostatic field; for the interstellar plasma, these effects could be significant for frequencies of a few tens of inverse seconds and below. Also, the effects of resonance regions on propagation should be investigated in more detail than here.

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